Government of **Western Australia**
School Curriculum and Standards Authority

scsa

ATAR course examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST

Section One: Calculator-free

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

WA student number: In figures

In words _____

_____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters
Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	49	35
Section Two: Calculator-assumed	11	11	100	92	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2021: Part II Examinations*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Section One: Calculator-free 35% (49 marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 (4 marks)

The Argand diagram below shows the complex numbers z and $z+2$ where $z = cis\left(\dfrac{\pi}{3}\right)$.
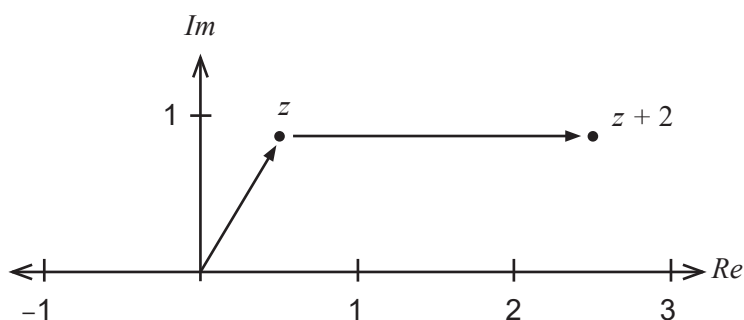
Determine the exact value for:

(a) $Arg(-z)$. (1 mark)

(b) $|z+2|$. (3 marks)

Question 2 **(11 marks)**

The graphs of functions f and g are shown below.
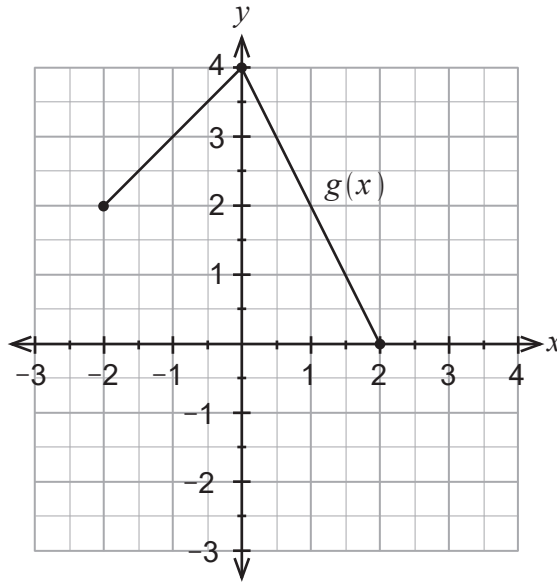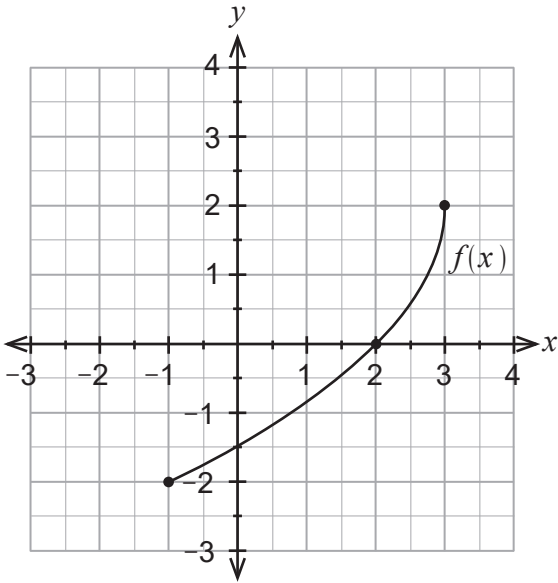
(a) Sketch the graph of function f^{-1} on the same axes used for function f. (2 marks)

(b) Explain why the inverse of g is not a function. (1 mark)

The defining rule for function f is $f(x) = 2 - 2\sqrt{3-x}$ where $-1 \leq x \leq 3$.

(c) Determine the rule for $y = f^{-1}(x)$. (3 marks)

(d)   Determine the exact value for $g\left(f\left(0\right)\right)$.   (2 marks)

(e)   Determine the domain for the function $y = f\left(g(x)\right)$. Justify your answer.   (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 3   **(5 marks)**

Using an appropriate substitution, determine the exact value for $\displaystyle\int_{2}^{3} 15x\sqrt{x-2}\,dx$.

Question 4 **(5 marks)**

Consider the function $f(x) = \dfrac{x^2 - 4}{x + 1} = x - 1 - \dfrac{3}{x + 1}$.

Sketch the graph of the function $y = f(x)$ on the axes below. Indicate clearly the x and y intercepts and any asymptotes.
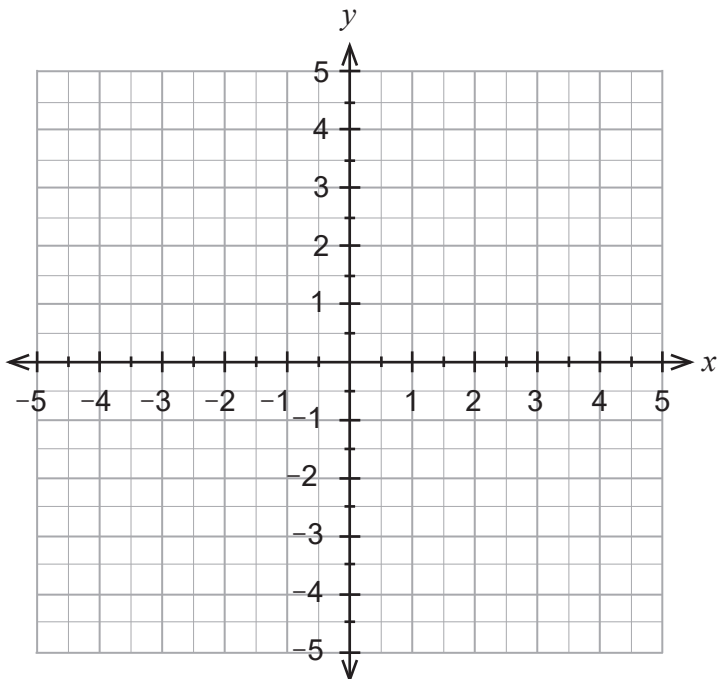
A spare grid is provided at the end of this Question/Answer booklet. If you need to use it, cross out this attempt and indicate that you have redrawn it on the spare grid.

See next page

Question 5   **(5 marks)**

(a)   Given that $\dfrac{7x^2 - 12x + 2}{(x-2)(x^2+2)} = \dfrac{a}{x-2} + \dfrac{bx}{x^2+2}$ determine the values of a and b.

                                                            (2 marks)

(b)   Hence determine $\int \dfrac{7x^2 - 12x + 2}{(x-2)(x^2+2)}\,dx$.   (3 marks)

Question 6   **(5 marks)**

Consider the quartic polynomial $P(z) = z^4 - 6z^3 + 31z^2 - 52z + 60$.

(a) Given that $P(2+4i) = 0$, determine a quadratic factor of $P(z)$.   (2 marks)

(b) Hence solve the equation $z^4 - 6z^3 + 31z^2 - 52z + 60 = 0$.   (3 marks)

Question 7   **(5 marks)**

The number 2021 can be expressed as a product of two consecutive prime numbers: $43 \times 47 = 2021$.

Consider the complex equation $z^{43} = 1$.

(a)   Write an expression for the roots of $z^{43} = 1$.   (2 marks)

Let w be any one of the roots of the equation $z^{43} = 1$.

(b)   How many of these roots will also be a solution of the equation $z^{47} = 1$?
            Justify your answer.   (3 marks)

Question 8 **(9 marks)**

The heart-shaped figure shown is given by the equation $x^2 + \left(y - \sqrt{|x|}\right)^2 = 2$.

For $x \geq 0$, this equation becomes $x^2 + \left(y - \sqrt{x}\right)^2 = 2$. The curve $y = \sqrt{x}$ is also drawn.
This heart-shaped curve has the special property that for each x coordinate in its domain its two y coordinates are an equal vertical distance from the curve $y = \sqrt{x}$.
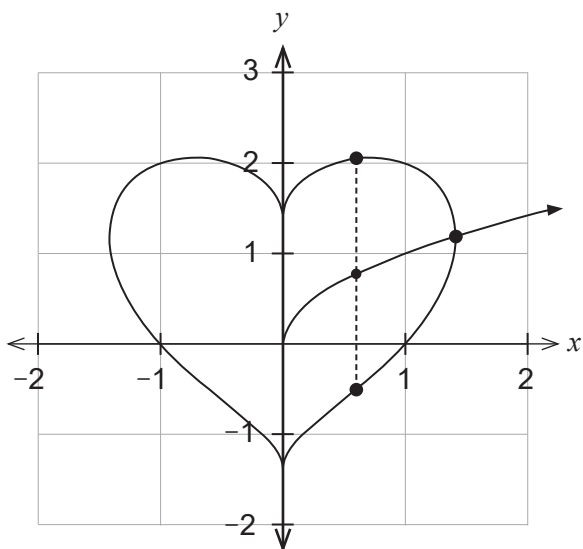
(a) Explain why the domain for the curve given by $x^2 + \left(y - \sqrt{x}\right)^2 = 2$ is $0 \leq x \leq \sqrt{2}$.

(2 marks)

(b) Show that the total area enclosed by the heart-shaped figure is given by:

$$Area = 4\int_0^{\sqrt{2}} \sqrt{2 - x^2}\, dx.$$

(2 marks)

(c)   By using the substitution $x = \sqrt{2}\sin\theta,$ evaluate the total area enclosed by the heart-shaped figure, and hence see why it can be said that 'π is at the heart of mathematics'. (5 marks)

End of section

Supplementary page

Question number: _____

Spare grid for Question 4.